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Remember to study

1. Exam 1 and Review Packet
2. Exam 2 and Review Packet
3. Exam 3 and Review Packet
4. The material since Exam 3
5. Homework and solutions

Optimization Worksheet Math 261

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Optimization

1. **Define:** We say that f has a *critical point* at (a, b) if

   In other words, either ____________________ or ____________________

2. The ____________________ are all the *potential* local maxima and minima.

3. **Second Derivative Test:** Suppose that f has a critical point at (a, b) and if f is differentiable on a disc around (a, b).

   $$\text{Define } D = f_{xx}(a,b)f_{yy}(a,b) - \big(f_{xy}(a,b)\big)^2$$

   There are four cases

   (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then ____________________

   (b) If $D > 0$ and $f_{xx}(a, b) < 0$, then ____________________

   (c) If $D < 0$, then ____________________

   (d) If $D = 0$ then ____________________

4. To apply the second derivative test to multiple critical points, create the table

critical point	f_{xx}	f_{yy}	f_{xy}	$D = f_{xx}fyy - (f_{xy})^2$	analysis

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5. Find all critical points of the given function and use the second derivative test to identify each as a local maximum, a local minimum, or undetermined.

$$f(x, y) = x^2y - 4y$$

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6. Find all critical points of the given function and use the second derivative test to identify each as a local maximum, a local minimum, or undetermined.

$$f(x, y) = x^2 + y^3 + 6x - 3y$$

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7. Find all critical points of the given function and use the second derivative test to identify each as a local maximum, a local minimum, or undetermined.

$$f(x, y) = -x^2 + xy - y^2 + 12y$$

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Integrating Functions of Two Variables

Notation: we denote rectangles as the *cartesian* product of two intervals.

$$R = [a, b] \times [c, d] = \{(x, y) : a \leq x \leq b \text{ and } c \leq y \leq d\}$$

1. Let $R = [0, 3] \times [0, 2]$. Compute the following integral

$$\iint_R xy^3 \; dA$$

2. Let $R = [1, 2] \times [0, \frac{\pi}{3}]$. Compute the following integral

$$\iint_R x \cos(2y) \; dA$$

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3. Let $\int_1^2 \int_y^{2y-1} 2x - y^2 \; dx \; dy$

   (a) Sketch the region of integration

   (b) Compute the specified volume.

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4. Let $f(x,y) = 2y - x^2$. Compute the net volume under f on the region D bounded between $y = 3x$ and $y = x^2$.

   (a) Sketch the region of integration

   (b) Write the volume under f on D as a Type I integral (where functions give the boundary for the top and bottom of D).

   (c) Compute the volume under $f(x,y)$ on the specified region

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5. Let $\int_0^4 \int_0^{\sqrt{x}} xy \, dy \, dx$

   (a) Sketch the region of integration

   (b) Compute the specified volume.

   (c) Write an *equivalent* integral with the order of integration reversed

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Integration in Polar Coordinates

1. Sketch the region defined by the polar rectangle $\mathcal{R}$ with $r \in [1,2]$ and $\theta \in [0,\pi/2]$. Then compute the net volume under $f(x,y) = 4xy + 3x$ on $\mathcal{R}$.

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2. Suppose that a washer $\mathcal{D}$ with an inner radius of 1m and an outer radius of 2m is centered at $\mathcal{O}$. The density of $\mathcal{D}$ is given by $d(x, y) = x^2 + y^2$. Compute the mass of the washer.

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Triple Integrals

1. Compute $\iiint_{\mathcal{B}} \frac{2x+6y}{z}\, dV$ on the box $\mathcal{B} = [0,3] \times [1,2] \times [1,e]$.

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2. Let $f(x, y, z) = x$ be a density function. Compute the mass of the region bounded by the planes $z = x - 2y$ and $z = 2x + y$ above the rectangle $[0, 2] \times [0, 1]$.

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3. Compute the integral
$$\int_0^2 \int_0^{1-x} \int_x^{x+y} x \;\, dz\, dy\, dx$$

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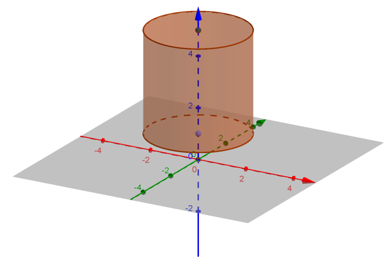
Integration in Cylindrical and Spherical Coordinates

1. A certain density function is given by $f(x, y, z) = z\sqrt{x^2 + y^2}$ in kg/m^3.

   Integrate f over the cylinder $\mathcal{W}$ with $x^2 + y^2 \leq 4$ and $1 \leq z \leq 5$.



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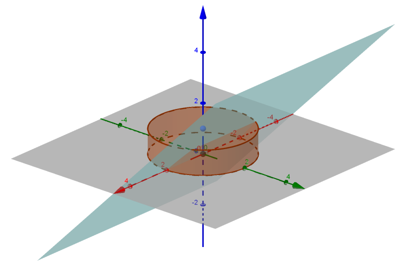
2. Set up and evaluate the integral of $f(x, y, z) = z$ on the cylinder $\mathcal{D}$ with $x^2 + y^2 \leq 4$ above the $x - y$ plane and below the plane $z = y$.



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3. Use integration with spherical coordinates to compute the volume of a sphere of radius r.

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4. Suppose you buy a spherical bearing with radius $\rho = 2\ m$. The bearing's density is given by $f(x, y, z) = x^2 + y^2 + z^2$ in kg/m^3. Find the mass of the bearing.